Using GOAL to Study for Exam CS2!

Actuarial University

QUESTION 5 OF 5 | Question # | Go! | Prev | Next

Quickly access the Hub for additional learning.

Flag problems for review, record notes, and email a professor for help.

Reveal answers and review to score yourself.

Written Answer

This problem features a written answer component. Please write out your solution using your preferred method. When you are ready to review your work and score yourself, click the following button.

Reveal / Review Written Answer Scoring

Written answers must be graded if you wish to include them in your GOAL Score during Practice sessions.

Information

The 'MortalityData1921' file contains mortality data for a particular population of females in 1921 from the Human Mortality Database. It includes three variables:

Age	Age, x, in single years from 25 to 90
d_x	Number of deaths aged x nearest birthday, d_x
Exposed_x	Exposed to risk at age x nearest birthday, E_x^c

Before answering this question, the MortalityData1921' file should be loaded into R and assigned to a data frame called *mortalitydata*.

Background information in the same format you'd see on your exam.

Question - Part A — Difficulty: Mastery

View difficulty level.

[5 points] (i) Create a new variable, mu_x, which represents the crude estimate of μ_x for each age x. Then, add this variable to the mortalitydata. Finally, plot the crude estimate of $m u_x$ against age x using a line graph.

Question - Part B — Difficulty: Mastery

[2 points] (ii) Comment on the age pattern of the crude values of μ_x shown in the graph in part (i).

Question - Part C — Difficulty: Mastery

[7 points] (iii) Perform a graduation of the crude values of μ_x using the formula:

$$\mu_x = \alpha_1 \exp(\alpha_2 x + \alpha_3 x^2)$$

.

Specify the values of α_1, α_2, and α_3 in your answer script.

Exam style questions mirroring IFoA points system for easy scoring.

Question - Part D — Difficulty: Mastery

[3 points] (iii) Overlay the line graph of graduated estimates of $m u_x$ against age x on the plot of the crude estimate of $m u_x$ against age x.

Question - Part E — Difficulty: Mastery

[9 points] (iv) Perform a chi-square goodness-of-fit test of the graduated values of μ_x from part (iii) using a 95% confidence level.

Question - Part F — Difficulty: Mastery

[6 points] (v) Perform a signs test of the graduated values of μ_x from part (iii) using a 95%-confidence level.

Not sure where to begin? Some questions feature "Help Me Start" to guide you in the right direction.

Help Me Start

See the solution below!

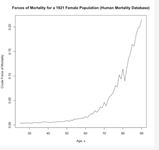
Question - Part A — Difficulty: Mastery

[5 points] (i) Create a new variable, mu_x, which represents the crude estimate of μ_x for each age x. Then, add this variable to the mortalitydata. Finally, plot the crude estimate of $m u_x$ against age x using a line graph.

Self Assessment & Solution - Part A

```
mortalitydata<-read.csv("MortalityData1921.csv", header=T,sep=",",stringsAsFactors=FALSE) [1 point]
mortalitydata$mu_x<-mortalitydata$d_x/mortalitydata$Exposed_x [2 point]

plot(mortalitydata$Age,mortalitydata$mu_x, type = "l", xlab = "Age, x", ylab = "Crude Force of Mortality
  main = "Forces of Mortality for a 1921 Female Population (Human Mortality Database)") [2 point]
```



Dive into the solutions and have each question broken down step by step.

Self Assessment Score: 4.00

Grade your performance using the self assesment.

Question - Part B — Difficulty: Mastery

[2 points] (ii) Comment on the age pattern of the crude values of μ_x shown in the graph in part (i).

Self Assessment & Solution - Part B

The values of mu_x increase exponentially with age and exhibit some roughness, particularly between 60 and 85 years, requiring graduation before use. **[2 point]**

Dive into the solutions and have each question broken down step by step.

Self Assessment Score: 2.00

Grade your performance using the self assesment.

Question - Part C — Difficulty: Mastery

[7 points] (iii) Perform a graduation of the crude values of μ_x using the formula:

$$\mu_x = \alpha_1 \exp(\alpha_2 x + \alpha_3 x^2)$$

.

Specify the values of α_1, α_2, and α_3 in your answer script.

Self Assessment & Solution - Part C

```
Model1 = lm( log(mu_x) ~ Age+I(Age^2), data = mortalitydata) [3 point]
summary(Model1)
Call:
lm(formula = log(mu_x) ~ Age + I(Age^2), data = mortalitydata)
Residuals:
     Min       1Q   Median       3Q      Max
-0.286875 -0.097393  0.007332  0.083236  0.271171
Coefficients:
             Estimate   Std. Error  t value  Pr(>|t|)
(Intercept) -5.678e+00  1.548e-01   -36.640   < 2e-16 ***
Age          -1.797e-02  5.768e-03    -3.115    0.00276 **
I(Age^2)      7.431e-04   4.962e-05    14.977   < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.1308 on 63 degrees of freedom
Multiple R-squared:  0.9985, Adjusted R-squared:  0.9982
F-statistic:  3301 on 2 and 63 DF,  p-value: < 2.2e-16

exp(as.numeric(coef(Model1)[1])) [1 point]
as.numeric(coef(Model1)[2]) [1 point]
as.numeric(coef(Model1)[3]) [1 point]
```

Hence: $\alpha_1 = 0.003447308$, $\alpha_2 = -0.017970288$, and $\alpha_3 = 0.0007431369$ [1 point]

Dive into the solutions and have each question broken down step by step.

Self Assessment Score: 7.00

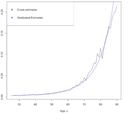
Grade your performance using the self assesment.

Question - Part D — Difficulty: Mastery

[3 points] (iii) Overlay the line graph of graduated estimates of $m u_x$ against age x on the plot of the crude estimate of $m u_x$ against age x.

Self Assessment & Solution - Part D

```
plot(mortalitydata$Age,mortalitydata$mu_x, type = "l", xlab = "Age, x", ylab = "Crude Force of Mortality
main = "Forces of Mortality for a 1921 Female Population (Human Mortality Database)") [2 point]
lines(mortalitydata$Age,mortalitydata$fittedmu_x, col = c("blue")) legend("topleft", legend = c("Crude e
```



Dive into the solutions and have each question broken down step by step.

Self Assessment Score: 2.75

Grade your performance using the self assesment.

Question - Part E — Difficulty: Mastery

[9 points] (iv) Perform a chi-square goodness-of-fit test of the graduated values of μ_x from part (iii) using a 95% confidence level.

Self Assessment & Solution - Part E

The null hypothesis is that the formula graduated rates are the true rates underlying the observed data **[0.5 point]**

The alternative hypothesis is that the formula graduated rates are NOT the true rates underlying the observed data **[0.5 point]**

```
mortalitydata$fittedmu_x = exp(fitted(Model1))
mortalitydata$zx = ((mortalitydata$d_x)-(mortalitydata$fittedmu_x * mortalitydata$Exposed_x)) / sqrt(mortalityd
mortalitydata$Exposed_x)
mortalitydata$zxsquared = (mortalitydata$zx)^2
chisquare = sum(mortalitydata$zxsquared);
chisquare [4 point]
```

```
[1] 40.19427
```

The test statistic has a chi-square distribution with m degrees of freedom, where m is the number of age groups less one for each parameter fitted. So, in this case m = 66 − 3 = 63.**[1 point]**

```
qchisq(0.95, 63) [1 point]
```

```
[1] 82.52873
```

Since 40.19427 < critical value

we fail to reject the null hypothesis at the 5% level. **[2 point]**

Dive into the solutions and have each question broken down step by step.

Self Assessment Score: 9.00

Grade your performance using the self assesment.

Question - Part F — Difficulty: Mastery

[6 points] (v) Perform a signs test of the graduated values of μ_x from part (iii) using a 95% confidence level.

Self Assessment & Solution - Part F

The signs test is a test for the overall bias. Define the test statistic:

$$\mathrm{P} = \text{ Number of } z_x \text{ that are positive.}$$

Under the null hypothesis, $\mathrm{P} \sim \mathrm{Binomial}(m, 1/2)$. An excess of either negative or positive deviations is a defect, so we apply a two-tailed test. If the number of age groups is large (more than 20), we can use the approximation:

$$\mathrm{P} \sim \mathrm{Normal}(1/2m, 1/4m).$$

From our data, the test statistic is:

```
sum((mortalitydata$zx)>0) [2 point]
[1] 34
and the critical value is:
qnorm(0.975, 66/2,66/4) [2 point]
[1] 65.33941
```

Since 34 < critical value

we fail to reject the hull hypothesis at the 5% level. **[2 point]**

Dive into the solutions and have each question broken down step by step.

Self Assessment Score: 15.00

Grade your performance using the self assesment.

Rate this problem: Excellent | Needs Improvement | Inadequate

Rate a problem or give feedback.